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Adaptive-critic based optimal neuro control synthesis for distributed parameter systems☆

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Abstract

A neural network based optimal control synthesis approach is presented for systems modeled by partial differential equations. The problem is formulated via discrete dynamic programming and the necessary conditions of optimality are derived. For synthesis of the controller, we propose two sets of neural networks: the set of action networks captures the mapping between the state and control, while the set of critic networks captures the mapping between the state and costate. We illustrate the solution process with a parabolic equation involving a nonlinear term. For comparison, we consider the linear quadratic regulator problem for the diffusion equation, for which the Ricatti-operator based solution is known. Results show that this adaptive-critic based systematic approach holds promise for obtaining the optimal control design of both linear and nonlinear distributed parameter systems. © 2001 Published by Elsevier Science Ltd.

Keywords: Distributed parameter system; Optimal control; Neural control; Adaptive critic

1. Introduction

The control of distributed parameter systems is a topic that has been well studied from both a mathematical and an engineering point of view. There is a large and growing literature of theoretical results on methods for the control of partial differential equations (PDEs). These methods start with an infinite-dimensional model and as such are able to address mathematical issues such as well-posedness, model robustness and approximation issues with regard to simulation (see, e.g., Balas, 1998; Banks, Smith, & Wang, 1996). While there are many benefits to starting with an infinite-dimensional model, even a complete analysis of the dynamics, as exists for many controlled systems described by PDEs, may fail to adequately address the difficult problems of calculating and implementing the control law, especially in the case of nonlinear systems. In the special case of linear quadratic regulator (LQR) problems an explicit formula for the optimal control may be obtained via an operator Ricatti equation, although such formulas are limited to linear systems and implementing a control law expressed via an infinite-dimensional operator can be difficult.

One of the benefits of a neural network approach, in the control of lumped parameter systems, has been the ability to control nonlinear systems. Indeed, neural network synthesis of nonlinear control laws has been used in applications ranging from balancing an inverted pendulum to robotic control. For a good survey of the neural network based controls, we cite Hunt, Sbarbaro, Zbikowski, and Gawthrop (1992). Controllers based on the neural network design have been used to address optimal control and decision problems in noisy, nonlinear, non-stationary environments (White & Sofge, 1992; Narendra & Parthasarathy, 1990). This approach can also be used to obtain optimal control in feedback form (Edwards & Goh, 1995; Plummer, 1996; Nakanishi, Kohda, & Inoue, 1997; Balakrishnan & Biega, 1996).

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Unlike linear systems, the stability of nonlinear systems can be highly sensitive to initial conditions. If one wishes to control a system for which the initial conditions are not completely specified or known, or if the system is sensitive to variations in the initial conditions, it is necessary to obtain control functions that apply to an entire range of initial conditions. The method of dynamic programming (see, e.g., Bryson & Ho, 1975) handles this by producing a family of optimal paths, or what is known as the "field of extremals". One great drawback of this approach is that it requires a prohibitive amount of computation and storage requirements. Hence, unless the problem is simple, controller synthesis based on the dynamic programming, although of theoretical interest, is not feasible in practice.

The idea of using a neural network as a critic network was originally proposed by Widrow, Gupta, and Maitra (1983). An adaptive critic system, consisting of two adaptive elements or neurons, was proposed by Barto, Sutton, and Anderson (1983), which was mainly a reinforcement learning technique. A model-based approximate (discrete) dynamic programming approach embedded in an adaptive-critic technique was proposed in Balakrishnan and Biega (1996), which makes it possible to synthesize the closed-loop optimal controllers for complex lumped parameter systems, without becoming computationally overwhelmed. The technique has successfully been applied to complex lumped parameter problems producing state feedback optimal controllers for an envelope of initial conditions. Interesting articles on the philosophical justification for adaptive-critic structures was written by Werbos (1992, 1996).

Advantages of the adaptive-critic neural network approach include the fact that no a priori assumptions about the form of the feedback control are needed; e.g., one need not assume the control is a known function of the state. Although the technique does not result in an analytic expression for the control law (such as a feedback operator for the LQR problem obtained from a Ricatti equation) this method does not require any approximation (like linearization) of the system dynamics in order to synthesize the control. This method iteratively approximates and optimizes a control law during off-line "training" (network synthesis) in which an entire envelope of states is considered. One consequence of this method is that the resulting control is available to be used as on-line state-feedback control. Moreover, the methodology can be applied to synthesize optimal controllers for linear and nonlinear systems, following the same steps. The adaptive-critic based computational tool presented in this paper, in a similar perspective, provides a straight forward method for constructing state-feedback control of PDE-based models, for an entire envelope of initial state profiles. The approach allows the philosophy of dynamic programming to be carried out in a distributed parameter framework, without the need for impossible computation and storage requirements.

This paper is a first step in using the adaptive-critic neural network method for optimal control of distributed parameter systems. This is described in detail in Section 3. In Section 2, an approximate dynamic programming formulation for the optimal control of a PDE-based model is presented and the necessary equations are derived. We point out that the necessary conditions of optimality derived in Section 2 and the controller synthesis approach can be generalized to a system of PDEs, although only the scalar state and control case is addressed here. In Section 4, we apply the method to some relatively simple partial differential equations in one spatial dimension. We first apply it to a nonlinear system obtaining results that match reasonably with the numerical results obtained (for a single, fixed initial condition) by Sage and Chaudhuri (1967). In order to compare the results of the adaptive-critic method to a true analytic solution, we consider the infinite-time LQR problem for a linear heat equation and compare the result to the known analytic solution obtained via linear operator/spectral methods. Conclusions are presented in Section 5.

2. Optimality conditions: approximate dynamic programming

In this section we pose the optimal control problem of a PDE-based model in an approximate (discrete) dynamic programming format and derive the necessary condition for optimality. We also develop the propagation equations for the costates associated with optimal control. The general problem is that of finding the control $u(t,y)$, $y_0 \leqslant y \leqslant y_f$, $t_0 \leqslant t$, that minimizes an infinite-time performance index $J = \int_{t_0}^{t_f}\int_{y_0}^{y_f} \Psi(t,y,x,u)\,\mathrm{d}y\,\mathrm{d}t$ for a state $x(t,y)$ which satisfies a PDE $\partial x/\partial t = f(t,y,x,\partial x/\partial y,\partial^2 x/\partial y^2,\ldots;u)$, along with appropriate boundary conditions.

2.1. The discrete problem

The discretized system dynamics considered in this paper are expressed as an explicit finite-difference scheme

$$x_{k+1,j} = f_k(x_{k,1},\ldots,x_{k,M},u_{k,j}),\quad k=1,\ldots,N,\ j=1,\ldots,M, \tag{1}$$

where the first subscript k denotes the time step and the second subscript j represents the spatially discretized nodal number.

We consider a general cost function, to be minimized, of the form

$$J = \sum_{k=1}^{N-1}\sum_{j=1}^{M} \Psi_{k,j}(x_{k,j},u_{k,j}), \tag{2}$$